

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name : Engineering Mathematics-I

Subject Code : 4TE01EMT2

Branch : B.Tech (All)

Semester : 1

Date :21/04/2016

Time :10:30 To 1:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1

Attempt the following questions:

(14)

- a) The polar form of complex number $\frac{1+i}{1-i}$.
- a) $\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$ b) $\sin \frac{\pi}{2} + i \cos \frac{\pi}{2}$ c) $\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}$ d) $\sin \frac{\pi}{4} + i \cos \frac{\pi}{4}$
- b) The Imaginary part of Complex number e^{3z} is
- a) $e^y \sin x$ b) $e^x \cos y$ c) $e^{3x} \cos 3y$ d) $e^{3x} \sin 3y$
- c) $\lim_{x \rightarrow 0} \frac{\tan x}{x} = \underline{\hspace{2cm}}$.
- a) 0 b) 1 c) ∞ d) -1
- d) $\lim_{x \rightarrow 0} \frac{x - \sin x}{x} = \underline{\hspace{2cm}}$.
- a) 0 b) 1 c) ∞ d) -1
- e) The rank of diagonal matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is
- a) 0 b) 1 c) 3 d) -2
- f) If the rank of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is 3, then x is not equal to
- a) 3 b) 4 c) 5 d) 6
- g) If the power of x & y both are even, then the curve is symmetrical about
- b) X-axis b) Y-axis c) about both X & Y axes d) None of these
- h) If the two tangents at the point are real & coincide, the double point is called
- a) a node b) a cusp c) a conjugate point d) None of these
- i) The series $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$ represents expansion of



- a) $\sin x$ b) $\cos x$ c) $\cosh x$ d) $\sinh x$
- j)** If $y = \cos^{-1} x$, then $x = \dots$
- a) $1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \dots$ b) $1 + \frac{y^2}{2!} + \frac{y^4}{4!} + \dots$ c) $y - \frac{y^3}{3!} + \frac{y^5}{5!} - \dots$ d) None of these
- k)** If $u = \sin^{-1} \left(\frac{x^2 + y^2}{x + y} \right)$ then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \dots$
- a) u b) $2u$ c) $\tan u$ d) $\sin u$
- l)** If $p = r \tan \theta$ then $\frac{\partial p}{\partial r}$ is
- a) $\tan \theta + r \sec^2 \theta$ b) $\sec^2 \theta$ c) $\tan \theta$ d) None of these
- m)** If $x = r \cos \theta, y = r \sin \theta, z = z$ then $\frac{\partial(x, y, z)}{\partial(r, \theta, z)} = \dots$
- a) $\frac{1}{r}$ b) $r^2 \sin \theta$ c) r d) $r^2 \cos \theta$
- n)** If $\frac{\partial(u, v)}{\partial(x, y)} * \frac{\partial(x, y)}{\partial(u, v)} = \dots$
- a) 0 b) -1 c) 1 d) None of these

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

A i) Find modulus and principal argument of $z = \frac{1-7i}{(3+4i)}$. **(03)**

ii) Simplify $\frac{(\cos 3\theta + i \sin 3\theta)^8 (\cos \theta - i \sin \theta)^5}{(\cos 2\theta + i \sin 2\theta)^{-2} (\cos 5\theta - i \sin 5\theta)^{-3}}$ **(04)**

B (i) Find and plot the fourth root of unity on the circle. **(07)**

Q-3 Attempt all questions (14)

A i) Evaluate: $\lim_{x \rightarrow 0} \left[\frac{1^x + 2^x + 3^x}{3} \right]^{\frac{1}{x}}$. **(04)**

ii) Find Maclaurin's Series of $f(x) = \sin x$. **(03)**

B i) Show that $f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2}; & (x, y) \neq (0, 0) \\ 0 & ; (x, y) = (0, 0) \end{cases}$ is continuous at every point **(04)**

except at the origin.

ii) Evaluate: $\lim_{x \rightarrow 0} (\cos x)^{\cot x} \dots$ **(03)**

Q-4 Attempt all questions (14)

A Trace the curve (Cissoid of Diocle) $y^2(2a - x) = x^3$. **(07)**



B If $y = \frac{x}{x^2 + a^2}$. find y_n . (07)

Q-5 Attempt all questions (14)

A Find the Eigen Values & Eigen Vectors of the Matrix $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$ (07)

B State Caley Hamilton Theorem . Verify Caley Hamilton Theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ & Hence, find A^{-1} (07)

Q-6 Attempt all questions (14)

A (i) Find the rank of matrix $A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \end{bmatrix}$ (03)

(ii) Find the inverse of the matrix $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ using Guass Jordan Method (04)

B $x + y + z = 6$ (04)

(i) Check the consistency of the system $x + 2y + 3z = 14$.

$$x + 4y + 9z = 36$$

(ii) Check the following set of vectors $(1,0,1), (1,1,0), (1,-1,1), (1,2,-3)$ is Linearly Independent or dependent ? (03)

Q-7 Attempt all questions (14)

A (i) If $u = \log(x^3 + y^3 - x^2y - xy^2)$. then Prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)^2 u = -\frac{4}{(x+y)^2}$. (05)

(ii) Find the values of $\frac{\partial f}{\partial x}$ & $\frac{\partial f}{\partial y}$ at the point $(4,-5)$. if $f(x, y) = x^2 + 3xy + y - 1$. (02)

B (i) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x} + \sqrt{y}}\right)$. Prove that (07)

a) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$

b) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{1}{4} (\tan^3 u - \tan u)$.

Q-8 Attempt all questions (14)

A (i) Find Maxima & Minima of the function $x^3 + y^3 - 3x - 12y + 20$. (05)



(ii) If $x = r \cos \theta, y = \sin \theta$ then find $\frac{\partial(x, y)}{\partial(r, \theta)}$.

B

(i) Find the Taylor's series expansion of $f(x) = \tan^{-1}\left(\frac{y}{x}\right)$ in powers of $(x-1)$

(02)

(05)

(02)

& $(y-1)$.

(ii) Find the equations of tangent plane & normal line at the point $(-2, 2, -3)$ to the

ellipsoid $\frac{x^2}{4} + y^2 + \frac{z^2}{9} - 3 = 0$

